

# Comparison Of Mathematics Textbook Problem Examples Based On Comprehensive Difficulty Model And Teaching Enlightenment

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## **Abstract:**

*Using the comprehensive difficulty model of mathematical example problems, the difficulty of example problems in the A version (2019) and Xiangjiao version (2019) of the preparatory knowledge topics is compared. The results show that the comprehensive difficulty of the example questions of the Xiangjiao version of the textbook is greater than that of the A version of the textbook. In terms of three factors: background, operation and knowledge content, the difficulty of the Xiangjiao version is higher than that of the A version; In terms of mathematical cognition and reasoning, the difficulty of the Xiangjiao version is lower than that of the A version. Based on this, three teaching suggestions are put forward for teachers to effectively use the new version of teaching materials and adapt to the teaching of the theme of preparatory knowledge.*

**Keywords:** *high school mathematics textbooks; Prerequisites; difficulty of example questions; compare*

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## **I. Introduction**

Since 2013, the Ministry of Education has launched a new round of basic education mathematics curriculum reform and developed a new "Mathematics Curriculum Standards for General High Schools" (revised in 2017 and revised in 2020). Major publishing houses have successively organized experts to compile new editions of high school mathematics textbooks. Mathematics textbooks are a model for students to engage in mathematics learning and teachers to engage in mathematics teaching, providing basic materials for mathematical activities and basic clues, basic contents and basic methods for students' mathematical activities [1]. Preparatory knowledge is a new theme in the high school mathematics curriculum standards, is one of the five major themes of the compulsory courses in the course standard, it includes sets, common logical terms, equality relations and inequality relations, from the functional point of view of univariate quadratic equations and monic quadratic inequalities and other learning content, is to help students smoothly from the compulsory education stage to high school mathematics stage of the transition knowledge, it requires students to learn the preparatory knowledge content, can adapt to the characteristics of high school mathematics learning style, can do a good job in learning psychology, Preparation in terms of learning style and knowledge and skills [2]. The theme of "Preparatory Knowledge" responds to the practical difficulties encountered by students in the transition from junior high school to high school. Some studies have pointed out that the characteristics of high school mathematics learning are quite different from those in junior high school, on the one hand, the difficulty, depth and breadth of course content

are significantly improved, which is challenging for students' learning, that is, mathematical language abstraction, thinking ability hierarchical, knowledge content enrichment, and knowledge application flexibility and integration; On the other hand, the poor learning state of some freshmen restricts their high school mathematics learning, such as students' excessive psychological dependence, lax thinking, neglect of the foundation, and inability to learn the law, etc. [3].

For the practical problems exposed above, teachers should correctly grasp the teaching orientation of preparatory knowledge in order to pave the way for students to adapt to high school mathematics learning. The teaching of preparatory knowledge is not to let students learn a few more theorems, but to help students use new mathematical language and tools, sort out the connection between new knowledge and old knowledge, and cultivate students' own independent learning ability in the process of connecting academic sections [4]. Therefore, teachers need to grasp the difficulty of the course content in order to make a good transition to junior and senior mathematics teaching and achieve a target. As an important part of the teaching materials, the selection and arrangement of example questions to a certain extent reflect the specific requirements for teaching content in the curriculum standards, reflect the writing style of the publishing house, and have the functions of demonstrating and leading, revealing methods, introducing new knowledge, consolidating new knowledge, thinking training and cultural education [5]. The difficulty of the example questions also reflects the difficulty of the textbook to some extent [6]. Taking the difficulty of example problems in the study of preparatory knowledge as the starting point, the key points of junior and senior high school mathematics bridging teaching will be discussed, which will help teachers grasp the appropriateness and difficulty of the content of the textbook, better carry out the classroom teaching design, implementation and evaluation under the new textbook, promote students to gradually adapt to the characteristics of high school mathematics learning, and comprehensively improve the core literacy of students' mathematics.

## II. Research Objects And Methods

Chapters 1 and 2 of the compulsory first volume of the Renjiao Edition of "General High School Textbook Mathematics (A Edition)" (2019 Edition) and the contents of the first chapter and Chapter 2 of the first volume of the Xiangjiao Edition of "General High School Textbook Mathematics (2019 Edition)" [7][8], hereinafter referred to as "Renjiao A Edition" and "Xiangjiao Edition", and use comparative and quantitative research methods to analyze the examples of the selected content. The comprehensive difficulty model proposed by Professor Wang Jianpan and Professor Bao Jiansheng [6] is an important method in the field of comparative research of mathematics textbooks, which promotes the development of mathematics textbook research from qualitative research to quantitative research [9]. In this study, with the help of Pu Anshan fine-tuned comprehensive difficulty model of example problems, the difficulty of the example problems in the two versions of the textbook is compared from the five difficulty factors of background, mathematical cognition, operation, reasoning and knowledge content, and each difficulty factor is divided into different level levels, as follows.

$$d_i = \frac{\sum_{j=1} n_{ij} d_{ij}}{n} \left( \sum_{j=1} n_{ij} = n, i = 1,2,3,4,5; j = 1,2, \dots \right)$$

where  $d_i$  represents the weighted average of the  $i$ th difficulty factor,  $d_{ij}$  represents the weight of the  $j$ th level of the  $i$ th difficulty factor, and using the rank weight, then  $d_{ij} = j$ ,  $n_{ij}$  indicates the number of questions at the  $j$ th level of the  $i$ th difficulty factor. In order to make the difficulty level data of the example problems as accurate as possible, the researchers and other mathematics education researchers first make separate judgments, discuss and reach a consensus when there are inconsistencies.

### III. Research Results And Analysis

**Table 1 Comparison of the difficulty factors of the two sets of textbook examples**

Factor	Difficulty	Renjiao A version		Xiangjiao version		Weighted average	
		Question volume	Percentage	Question volume	Percentage	Renjiao A version	Xiangjiao version
Background	No background	24	80%	34	80%	1.37	1.50
	Personal life	2	7%	1	2%		
	Public common sense	3	10%	1	2%		
	Scientific background	1	3%	6	14%		
Mathematical cognition	Operate	8	27%	12	29%	2.40	2.17
	Conception	6	20%	16	38%		
	Understand	12	40%	9	21%		
	Analyse	4	13%	5	12%		
Operation	No calculations	14	47%	8	19%	2.17	2.86
	Numerical operations	1	3%	4	10%		
	Simple symbolic operations	11	37%	16	38%		
	Complex symbolic operations	4	13%	14	33%		
Illation	No reasoning	5	17%	8	19%	2.13	2.02
	Simple reasoning	16	53%	25	60%		
	Complex reasoning	9	30%	9	21%		
Knowledge content	A point of knowledge	15	50%	19	45%	1.63	1.79
	Two knowledge points	11	37%	13	31%		
	Multiple knowledge points	4	13%	10	24%		

#### Analysis of background factors

As can be seen from Table 1, the proportion of personal life background and public common sense background in the example questions is higher than that of the Hunan version, and the proportion of scientific background is lower than that of the Hunan version. From the perspective of the content of the preparatory knowledge of the textbook, most of the knowledge content is the content that students have learned in junior high school, such as "the properties of equations and inequalities", "quadratic functions and univariate quadratic equations, inequalities" and other content, so the background of the example questions is mostly the background

of mathematical situations that students are familiar with. From the perspective of the old and new textbooks, the two versions of the new textbooks accounted for 20% of the example questions with background, and compared with the example questions background of the old textbooks, the type of example questions in the new textbooks was greatly improved [10]. This is in line with the idea of high school mathematics teaching to create appropriate situations, inspire students to think, and guide students to grasp the essence of mathematics content[2].

#### **Analysis of mathematical cognitive factors**

It can be seen from Table 1 that the proportion of Renjiao A version in terms of operation level and analysis level is roughly the same as that of Xiangjiao version, and the comprehension level is higher than that of Xiangjiao version, but lower than that of Xiangjiao version at the conceptual level. According to the existing research, the A version of the Renjiao textbook focuses on the application of mathematics and inherits the characteristics of the old version of the textbook [11]. It can be seen that both editions of the textbook pay attention to students' memorization of concepts, rules and forms of expression, pay attention to understanding the connotation of concepts, principles, laws and mathematical structures, pay attention to the variation of problems, pay attention to the reasonable selection of mathematical methods, and pay attention to the flexible use of knowledge. It reflects the basic concept of the general high school mathematics curriculum standards, highlights the internal logic and thinking methods of mathematics, and focuses on improving students' ability to apply mathematics to solve practical problems.

#### **Analysis of computing factors**

It can be seen from Table 1 that the proportion of numerical operations, simple symbolic operations and complex operation examples in the Renjiao (A) version of the textbook is lower than that of the Xiangjiao version, and the number of questions without operation is much higher than that of the Xiangjiao version. Most of the arithmetic level of the example problems of the A version of the textbook focus on no operation and simple symbolic operation, while most of the example problems of the Xiangjiao version of the textbook focus on simple symbolic operation and complex symbolic operation. The proportion of example problems that belong to numerical operations in both textbooks is less difficult than that of the other three types of operations. This is determined by the content composition characteristics of the prior knowledge. In the arrangement of preparatory knowledge content, the contents of sets and logical terms, equality relations and inequality relations, basic inequalities, etc., and unary quadratic equations from a functional point of view, etc., involve operations that are no longer numerical operations in junior high school, but more involve simple symbolic expressions and operations, which are more abstract perspectives for students to experience the characteristics of high school mathematics learning [11]. For example, in the content of inequalities, "relationship", "connection", "basic ideas and methods" and so on as keywords, so that students can understand the commonality and difference between equations and inequalities through analogy, master the properties of inequalities and the basic mathematical ideas and methods they contain, and experience the role of "operation" in the study of algebraic properties [12].

#### **Analysis of reasoning factors**

It can be seen from Table 1 that the proportion of questions without reasoning difficulty in the Renjiao (A) version of the textbook is comparable to that of the Xiangjiao version of the textbook, and the proportion of questions in the complex reasoning difficulty of the Renjiao (A) version of the textbook is more than that of the Xiangjiao version. Both textbooks focus on simple reasoning in reasoning factors, and at the same time, the amount of questions at the level of complex reasoning is the same. The simple reasoning and complex reasoning of example problems in the two editions of the textbook accounted for more than 81%, which shows that the two

sets of textbooks pay great attention to the application of mathematical logical reasoning in the process of example design and analysis. The curriculum standard arranges "preparatory knowledge", with "language" and "tools" as keywords, so that students can sort out the mathematical content they have learned in a new language expression in the learning of collection and common logical terms, and improve students' level of mathematical expression in abstract symbolic language by mastering some concise and accurate mathematical language, so as to improve the rigor and accuracy of mathematical reasoning [12]. The common logical terms used in the preparatory knowledge are a good material for cultivating students' logical reasoning literacy, so the setting of relevant example problems involves students' reflection and reconstruction of previous knowledge, and examines the existing knowledge content from a logical and accurate perspective to promote the structuring of knowledge.

### **Analysis of knowledge content factors**

Among the 30 example questions in the A-edition textbook, 50% of the examples of a single knowledge point, 37% of two knowledge points, and 13% of multiple knowledge points. Among the 42 example questions in the Xiangjiao textbook, a single knowledge point accounted for 45%, two knowledge points accounted for 31%, and multiple knowledge points accounted for 24%. It can be seen that the number of questions in the three difficulty levels of the knowledge content factor in the two versions of the textbook shows a downward trend, and the number of questions in multiple knowledge points is the least. Among the 30 example questions in the Renjiao (A) version of the textbook, the proportion of "example questions of a single knowledge point" and "two knowledge points" is higher than that of the Xiangjiao version, while the proportion of "multiple knowledge points" is lower than that of the Xiangjiao version.

### **Comprehensive difficulty analysis**

From the comparison table of the difficulty factors of the two sets of textbook examples in Table 1, it can be seen that: (1) In terms of background, operation and knowledge content, the difficulty of the mathematics preparatory knowledge examples of the Xiangjiao version of the textbook is higher than that of the A version of Renjiao, and in terms of mathematical cognition and reasoning, the difficulty of the mathematics preparatory knowledge examples of the A version of Renjiao is higher than that of the Xiangjiao version. (2) Among the 5 factors of the two sets of textbooks, the difficulty value of the background factor is much smaller than the other 4 factors, and the two sets of textbooks pay special attention to mathematical cognition, operation and reasoning, which is determined by the knowledge content and structural characteristics of the preparatory knowledge itself, and the set, basic inequality and logical terms in the textbook are a good carrier for cultivating students' mathematical abstraction, logical reasoning and mathematical operation literacy, including the use of logical terms to sort out the knowledge that has been learned in junior high school, and these knowledge backgrounds are based on mathematical knowledge itself. (3) The two sets of textbooks did not maintain the balance of the five factors, and the comprehensive difficulty pentagonal of the A version of the textbook was tilted to the right, towards mathematical cognition, while the Xiangjiao version was tilted to the right and downward, biased towards operation. (4) After calculation, the comprehensive difficulty of the example questions of the A version of the textbook is 1.94 (the average of the weighted average of the five difficulty factors), and the comprehensive difficulty of the example questions of the Xiangjiao version of the textbook is 2.07.

## **IV. Research Conclusions And Recommendations**

### **Conclusion**

After the comprehensive difficulty analysis of the mathematical example problems given in the two sets of textbooks, the following conclusions are drawn.

The comprehensive difficulty of the Xiangjiao version of the textbook is greater than that of the A version of the textbook. In terms of three factors: background, operation and knowledge content, the difficulty of the mathematics preparatory knowledge example problems of the Xiangjiao version of the textbook is higher than that of the A version of the human teaching, and in terms of mathematical cognition and reasoning, the difficulty of the mathematics preparatory knowledge example problems of the A version of the human teaching is higher than that of the Xiangjiao version. Among the five factors of the two textbooks, the difficulty value of the background factor is much smaller than that of the other four factors, and both textbooks pay special attention to mathematical cognition, operation and reasoning.

In terms of background factors, the background of the example questions in the two sets of textbooks was selected as the main body without background test questions, both of which accounted for 80%, and the proportion of background questions was greatly improved compared with the old version of the textbooks.

In terms of exploration factors, the proportion of Renjiao A version in the operation level and analysis level is roughly the same as that of Xiangjiao version, and the comprehension level is higher than that of Xiangjiao version, and the conceptual level is lower than that of Xiangjiao version, and both sets of textbooks attach great importance to the application of mathematical knowledge.

In terms of arithmetic factors, the proportion of numerical operations, simple symbolic operations and complex arithmetic example problems in the Renjiao (A) version of the textbook is lower than that of the Xiangjiao textbook, and the number of questions without operation is much higher than that of the Xiangjiao version; Most of the arithmetic level of the example problems of the A version of the textbook focus on no operation and simple symbolic operation, while most of the example problems of the Xiangjiao version of the textbook focus on simple symbolic operation and complex symbolic operation.

In terms of reasoning factors, both sets of textbooks focus on simple reasoning in reasoning factors, and the proportion of questions without reasoning difficulty in the Renjiao (A) version of the textbook is comparable to that of the Xiangjiao version of the textbook, and the proportion of questions in the difficult part of the Renjiao (A) version of the textbook is more than that of the Xiangjiao version. The amount of questions at the level of complex reasoning is the same in both editions. The example questions of the two editions of the textbook, simple reasoning and complex reasoning, accounted for more than 81%.

In terms of knowledge content factors. The number of questions in the three difficulty levels of the knowledge content factor in the two versions of the textbook showed a downward trend, and the number of questions in multiple knowledge points was the least. Among the 30 example questions in the A-edition of the textbook, the proportions of "example questions of a single knowledge point" and "two knowledge points" are higher than those of the Xiangjiao version, while the proportion of "multiple knowledge points" is lower than that of the Xiangjiao version.

### **Recommendations**

Based on the mathematical background of example problems, ask appropriate questions

The background of the example questions in the preparatory knowledge in both textbooks is mainly based on mathematical knowledge. Context and problems are one of the important dimensions to portray the core literacy of high school mathematics. In the teaching of example problems, it is necessary to base on the example problems with mathematical knowledge as the background, and use new mathematical expression tools and new mathematical ideas as the carrier to put forward new mathematical problems. In different contexts such as social life and scientific activities, it is necessary to guide students to ask questions that can reveal the essence of mathematics, so as to improve students' ability to find and ask questions.

Pay attention to the reasoning of example problems and develop the quality of thinking

Both textbooks focus on the arithmetic and reasoning of example problems. In the teaching of example problems, on the one hand, when cultivating students' arithmetic ability, teachers strive to reflect the gradual process from concrete to abstract, from constant to variable, from one-way thinking to reverse and multi-directional thinking, from law to arithmetic. On the other hand, give full play to the function of different forms of reasoning, and inspire students to analyze mathematical problems with the help of analogical reasoning and inductive reasoning, be good at grasping the key of the problem, and put forward mathematical conjectures; Use deductive reasoning, verify conclusions, apply mathematical language to express problems, promote students to develop the habit of expression and communication of "complete steps, sufficient reasons, and standardized expression", and ultimately promote the development of students' mathematical thinking.

Focus on the synthesis of knowledge of example problems to improve mathematical cognition

The proportion of multiple knowledge in the two versions of the textbook is small, and the preparatory knowledge should pay attention to the integration of knowledge. The integration of knowledge points should pay attention to both the horizontal connection of knowledge and the vertical connection of knowledge synthesis. When students face the intertwining and infiltration of multiple knowledge points, teachers guide students to look at old problems with new ideas and new perspectives, so that students can experience the integrity, connection and development of mathematics, and ultimately promote the improvement of students' mathematical cognitive level.

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